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Research Article

Periodic Model Of The Interaction Between A Predator With A Double Mutualist

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ABSTRACT

This is a study of the predator-prey model as a form of coexistence of different species of animals in the open nature, a model that simulates the process of growth and de-growth of species by means of a periodic non-autonomous system of differential equations with respect to time. Applying Flaquet's theory, the System is reduced to a System where the matrix of the linear part is constant and the coefficients of the non-linear part are periodic with respect to time; necessary and sufficient conditions are obtained for the convergence of total populations to optimal distributions of populations; giving analytical and computational examples that corroborate the theoretical results obtained.

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INTRODUCTION

The modeling of real life processes by means of systems of periodic differential equations in general is of great importance and even more so when these processes are cyclical; in this case, to simplify the study, Flaquet's theory is applied to transform the system and extract conclusions [3]. The problem of coexistence between different species in an open ecological space is addressed in [4], and [5], where the different types of coexistence and one can favor or hinder the development of the other.

Many types of biological systems have been modeled mathematically with the purpose of realizing a better study of the natural interaction that exists between different species; in particular the prey-predator model has a relevant position due to the applicability not only of biology where it practically governs the coexistence of different species in open space, but also because it can be applied in other areas including economics. Here, in addition to the highly publicized Lorka-Volterra models, we will analyze lesser known ones in addition to their qualitative study.

The model was discovered independently by Lotka and Volterra, and for this reason it is known as a model Lotka-Volterra or model predator-prey that describes the evolution of prey and predators very well when they are located in an isolated ecosystem. Nevertheless, we have to clarify that two distinct populations in the same environment have several ways of surviving, for example:

- Mutual competence, that is to say compete for the same food source, tend to cause the extinction of a population of them, and the other tends to take advantage of the maximum capacity of environmental resources.
- Interdependence, that is to say the two populations provide some food resources, live peacefully among them, and tend to a state of equilibrium.
- The law of the jungle, is to say a population survives depending on the abundance of natural resources, called prey; however, the other population lives depending on the populations of prey, called the predator. The two elements are composed by the prey-predator model.
- The parasitic life, where one species feed on the other without killing it, but which shaves its quality of life.

Ecological processes in general have periodic behavior; it has already been seen that in [11] that the manifestations between a species that represents the prey and another predator its development has a periodic form; appearing periods in which one or the other can grow, there are multiple other cases where this type of coexistence can be perceived, such are the cases; [5], [6], [15], [16], [17] and [18].

In [10] three species are considered, where one is a natural prey, the second is a predator prey and the third is a natural predator, here the different equilibrium points are determined, and a qualitative study is made in a neighborhood of these points.

The problem of a mutualist pair and a predator of one is dealt with in [9]; giving conclusions regarding the densities of the species in a sufficiently long time, where the simulation of the problem is made using a System of Differential Equations.

In [8] refers to the mathematical modeling of several processes between them, dealing with the Prey-Predator model, which includes the possibility of system integration that simulates this interaction between two species.

The prey-predator model has been extensively treated using different techniques, here it may be included,[7]. Another focus on the Lootka-Volterra model is presented in [1]. In the master's dissertation [8] a very exhaustive study of the prey-predator model is made.

The treatment that we will make in this case corresponds with other models presented in the researches of diseases, especially the case of sicklemia, quite treated and with a large number of already developed models; we will only mention some of these works, in[14], [15] and [16], the qualitative study of different models in autonomous and non-autonomous form of the formation of polymers in the blood is treated.

Following these ideas from these previous works here is simulated the interaction between two species being simplified the referred system to arrive at conclusions of this process of coexistence in the open nature.

In nature the most frequent is the competence between different species in the struggle for survival, appearing here the prey-predator model developed by Lotka 1924; Volterra, 1926; Gause, 1934; Kostitzin, 1939. [2].

Drawing on the work of Lotka, the models that consider the population classified by age groups have been developed, in order to solve the limitations of the models that treat all the individuals of the population identically.

One of the most commonly used classical mathematical models is the dynamic system consisting of two elements (usually two species of animals) interacting in such a way that one (predator) species feeds on the other (prey). A typical example is the system consisting of foxes and rabbits, but it can be transferred without loss of generality to any other context, for example, that formed by sellers and buyers applicable to the Economy.

Foxes feed on rabbits and grass rabbits that we assume will never run out. When there are many rabbits, the population of foxes will increase since food is abundant, but there will come a time when the rabbit population will decline as foxes are abundant. By not having the foxes, enough food their population will decrease, which will again favor the rabbit population. That is to say, if they produce cycles of growth and decrease of both to the populations. Is there a mathematical model that explains this periodic behavior?

On the other hand, in the second decade of the 20th century the Italian biologist Umberto D'Ancona studied and compiled data on catches of fish of some types in the Mediterranean, on the one hand, seals (sharks, rachis, etc.), and other fish that were eaten by the previous ones (sardines, anchovies, etc.), in other words, one prey (the edible fish) and the other predator (seals).

One of the first reasons he thought was related to the First World War. In fact, at that time the first great war developed and this forced less boats to go fishing, and therefore, by reducing the intensity of fishing, this caused an increase in the number of predatory fish (seals). However, this argument had a problem and it was also that the number of edible fish had increased. In fact, if the intensity of fishing is small, then this fact benefits the predators more than the prey. The pertinent question was why? Briefly, two questions were raised:

- . How to explain the cyclical behavior of the evolution of two populations, where one species feed on the other?
- . Why does a low catch intensity favor predator more than prey?

A detailed study of these types of systems is analyzed in the authors' work [5], which characterizes the behavior of the Lotka-Volterra systems under the hypothesis that the prey grows exponentially in the absence of predators and the predator disappears in absence of prey, studying the behavior of the trajectories in an environment of the equilibrium positions, one can perceive the existence of closed orbits due to the periodicity of the solutions.

Among the models of interaction between species the classic prey-predator model can be highlighted, whose mathematical formulation is composed of Malthusian models and the law of mass action. The analogy can be easily observed in epidemiological models. The prey-predator model also known as the Lotka-Volterra model has also been the starting point for the development of new techniques and mathematical theories.

Predation is a very fundamental type of interaction in nature, where predators catch prey for their food. We can imagine that this relationship is beneficial only to the predator, but from the ecological point of view this is important to regulate the population density of both prey and predator.

Predators remove individuals from the population, consuming them; the ease of catching the prey depends greatly on the size relationship between the prey population and the predator. The greater the population of prey, the greater the possibility of its capture.

Predation occurs when an organism kills and feeds on beings of another species; the animal that killed it is called a predator, which already fed on the prey. Predators are usually found in smaller quantities and have characteristics that favor prey capture; among these characteristics, we can mention the sharp claws, speed and agility [5].

II. MODEL FORMULATION:

In the study of the interaction of different species in nature lately, mathematical modeling has been very important, since this has allowed us to make predictions regarding the future behavior of this coexistence process, determining if any species could at any time be endangered. extinction; All of this is important, because this way you know when you can introduce some artificial element such as fishing or house as the case may be.

The predator-prey model simulates the interaction between species in an open ecological space, in which case we will treat the case of three species. We will consider that there are two mutualistic species. The first one has only feeding limitations or problems with space only with an excessive increase of their species. concentration; in the second mutualistic species with the first, it is the prey of the third; it is such that it depends strongly on these species and very insignificant on its concentration, the third species has the normal behavior of a predator of the second species, let us admit further that this relationship of coexistence during the process does not change. problem through a System of Differential Equations.

To make the simulation by means of a system of differential equations, we will consider the following notations with respect to each of the species,

-$\tilde{x}_1$ -Concentration total of the first species in the moment t .

-$\tilde{x}_2$ – Total prey concentration, mutualistic species of the first, in the moment t .

-$\tilde{x}_3$ - Total predator concentration of second species in time t .

In addition, $\bar{x}_1$, $\bar{x}_2$, and $\bar{x}_3$ the optimal concentration values for each species respectively. Here the variables will be introduced x_1, x_2, e x_3 definidas do seguinte jeito: $x_1 = \tilde{x}_1 - \bar{x}_1$, $x_2 = \tilde{x}_2 - \bar{x}_2$, and $x_3 = \tilde{x}_3 - \bar{x}_3$, so if $\bar{x}_1 \to 0$, $\bar{x}_2 \to 0$, and $\bar{x}_3 \to 0$ the following conditions would be met, $\tilde{x}_1 \to \bar{x}_1$, $\tilde{x}_2 \to \bar{x}_2$, and $\tilde{x}_3 \to \bar{x}_3$, which would constitute the main objective of this work.

In this way, the periodic model in general that simulates this process has the following form.

$$\begin{cases} \frac{dx_1}{dt} = a_1(t)x_1 + a_2x_1x_2 - a_3x_1^2 \\ \frac{dx_2}{dt} = b_1(t)x_1 - b_2(t)x_3 - b_3(t)x_2^2 \\ \frac{dx_3}{dt} = -c_1(t)x_3 + c_2(t)x_2x_3 \end{cases} \quad \textbf{(1)}$$

The parametric functions presented in the system have the following meaning,

$a_1(t)$: it is the growth coefficient of the first species in relation to its own concentration.

$a_2(t)$: it is the growth coefficient of the first species in relation to the concentration of the second, mutualistic with it.

$a_3(t)$: is the coefficient of degrowth of the first species in relation to its own concentration due to mutual competence.

b_1(t): the growth coefficient of the second species in relation to the concentration of the first, mutualistic with it.

$b_2(t)$: it is the coefficient of degrowth of the second species in relation to the concentration of the third, predator of that species.

$b_3(t)$: it is the coefficient of degrowth of the second species in relation to its own concentration due to mutual competence.

$c_1(t)$: it is the coefficient of decrease of the third species in relation to its own concentration.

$c_2(t)$: é the growth coefficient of the third species in relation to the concentration of second, prey of that species.

With the initial conditions $x_1(0) = x_{10}$, $x_2(0) = x_{20}$, $x_3(0) = x_{30}$; in the model we are considering that the coefficients are all positive.

The system (1) can be written in vector form as follows:

$$x' = A(t)x + X(t,x) \qquad \textbf{(2)}$$

Where $x = col(x_1, x_2, x_3)$, $X(t,x) = col(X_1, X_2, X_3)$ and A (t) is the matrix of the linear part,

$$A(t) = \begin{bmatrix} a_1(t) & 0 & 0 \\ b_1(t) & 0 & -b_2(t) \\ -c_1(t) & 0 & 0 \end{bmatrix}$$

The functions $X_i\,(t, x_1, x_2, x_3)(i = 1,2,3)$ are disturbances not inherent in the process that can cause changes at any given time; from a mathematical point of view, they are infinitesimals of a higher order in a neighborhood of origin, these functions admit the following development in series of powers,

$$X_i\,(t, x_1, x_2, x_3) = \sum_{|p|\geq 2} X_i^p(t) x_1^{p_1} x_2^{p_2} x_3^{p_3}$$

Suppose that the matrix $A\,(t)$ and the series coefficients are periodic of period ω, is to say if they satisfy the relationships,

$$A(t+\omega) = A(t) \text{ and } X_i^{(p)}(t+\omega) = X_i^{(p)}(t)$$

Be it the linear system,

$$x' = A(t)x \qquad \textbf{(3)}$$

That is to say the system coefficients are ω - periodic functions; the study of systems (2) with variable but periodic coefficients must be reduced, at least theoretically, in a system where the coefficients of the matrix of the linear part of the system are constant, [3]. This corresponds with Floket's theory, which gives the theoretical foundations for this type of transformation.

It will denote for $\emptyset$ to the fundamental matrix of the system (3), and by B a matrix related to the previous one which is a constant matrix, such that $\emptyset(t+\omega) = \emptyset(t)B$, such that there is a matrix R for which $R = \omega^{-1} lnB$, define the function $G(t) = \varphi(t)e^{-Rt}$. These expressions will be used to further demonstrate the fundamental result of this work.

Theorem1: The transformation of coordinates

$$x = G(t)y \qquad \textbf{(4)}$$

Reduces the system (2) in the system

$$y' = Ry + Y(t,y) \qquad \textbf{(5)}$$

Proof: Deriving the transformation (4) along the trajectory of the systems (2) and (5) we have that,

$$x' = \phi'(t)e^{-Rt} - R\phi(t)e^{-Rt}y + \phi(t)e^{-Rt}y'\,.$$

Since x is a solution of (2) and $\emptyset$ is a fundamental matrix of the system (3), replacing the corresponding expressions, we can write,

$$A(t)\phi(t)e^{-Rt}y + X(t,\phi(t)e^{-Rt}y) = A(t)\phi(t)e^{-Rt}y - R\phi(t)e^{-Rt}y + \phi(t)e^{-Rt}y'\,.$$

Reducing similar terms, we have,

$$X(t,\phi(t)e^{-Rt}y) = -R\phi(t)e^{-Rt}y + \phi(t)e^{-Rt}y'$$

Isolating y' in the previous expression, you must:

$$y' = Ry + \phi(t)^{-1} e^{Rt} X(t, \phi(t) e^{-Rt} y)$$

Making,

$$\phi(t)^{-1} e^{Rt} X(t, \phi(t) e^{-Rt} y) = Y(t, y)$$

It is concluded that,

$$y' = Ry + Y(t, y)$$

Thus, the theorem (1) is demonstrated

Eigenvalues μ_1, μ_2, μ_3 of the matrix B represent the multipliers of the system (2), these values being in general complex numbers and eigenvalues $\lambda_1, \lambda_2, \lambda_3$ matrix R of the system (5) are called the characteristic index of the system (2).

From the matrix R definition, it is obtained:

$$\lambda_i = \omega^{-1} ln\mu_i \ \ (i = 1,2,3)$$

By defining the logarithm of a complex number, we obtain that,

$$ln\mu_i = ln|\mu_i| + i(arg\mu_i + 2k\pi) \ , (i = 1,2,3)$$

Theorem2: The equilibrium position of the system (3) is asymptotically stable if and only if $0 < |\mu_i| < 1$ $(i = 1,2,3)$ and so $\operatorname{Re} \lambda_i < 0$ $(i = 1,2,3)$.

Proof: Because of theorem1, the system (2) is equivalent to the system (5) and the proper values of the matrix R are $\lambda_1, \lambda_2, \lambda_3$ where,

$$\lambda_i = \omega^{-1} \ln|\mu_i| \ (i = 1,2,3,4) \ ,$$

How do we have to, $0 < |\mu_i| < 1$ like this, $Re\lambda_i < 0$, $(i = 1,2,3)$ which completes the proof of theorem2 using the first approximation method.

Note:

1) Nothing can be concluded if $|\mu_i| = 1$ for some, $(i = 1,2,3)$ and the rest such that $0 < |\mu_j| < 1$ $(i \neq j)$ because there is some λ_i with $\operatorname{Re} \lambda_i = 0$ and the rest with negative real part, this is a doubtful case and the stability of the null solution has to be determined using the corresponding series.

2) It has no real meaning if $\mu_i = 0$ for some $(i = 1,2,3)$.

3) If $1 < |\mu_i| < \infty$ for some $(i = 1,2,3)$, so in this case $\operatorname{Re} \lambda_i > 0$ for some $(i = 1,2,3)$ and so the system (5) is unstable.

The following is an example of the system as the shape of the system (2) where the different elements shown above are indicated.

Example1: Suppose there is no transfer of toxin from the liver to the blood or from the blood to the liver, as well as from the blood to the kidneys; so a system like that could be the system,

$$\begin{cases} x_1' = (ncosnt + p)x_1 + X_1(t, x) \\ x_2' = (ncosnt + p)x_1 - (nsennt + q)x_3 + X_2\,(t, x) \\ x_3' = -(nsennt + q)x_3 + X_3(t, x) \end{cases}$$

In that system you must,

$$A(t) = \begin{bmatrix} ncosnt + p & 0 & 0 \\ ncosnt + p & 0 & -(nsennt + q) \\ 0 & 0 & -(nsennt + q) \end{bmatrix}$$

This matrix satisfies the relationship,

$$A(t + 2\pi) = A(t)$$

This means that it is a periodic period matrix 2π. In this case, the fundamental matrix of the corresponding homogeneous system must be

$$\phi(t) = \begin{bmatrix} e^{sennt+pt} & 0 & 0 \\ e^{sennt+pt} & 0 & e^{cosnt-qt} \\ 0 & 0 & e^{cosnt-qt} \end{bmatrix}$$

This matrix is such that,

$$\phi(t + 2\pi) = \phi(t)B$$

Being B the following matrix,

$$B = \begin{bmatrix} e^{p\omega} & 0 & 0 \\ e^{p\omega} & 0 & e^{-q\omega} \\ 0 & 0 & e^{-q\omega} \end{bmatrix}$$

In this example, multipliers are $\mu_1 = 0, \mu_2 = e^{p\omega}, \mu_3 = e^{-q\omega}$ thus, in this example it has no real sense as it has a zero value of its own.

Example2: Be the system,
BBB

$$\begin{cases} x_1' = (cost - 1)x_1 + costx_1x_2^2x_3^2 \\ x_2' = \left(cost - \dfrac{1}{2}\right)x_1 - (sent + 2)x_3 - sen2tx_1^2x_2 \\ x_3' = -(sennt + 3)x_3 + x_1^2x_2^2x_3 \end{cases}$$

This system satisfies the conditions indicated above.

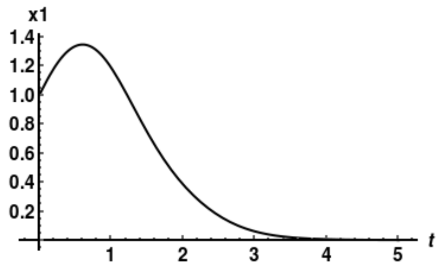

Fig.1: Graph ofx_1 against t.

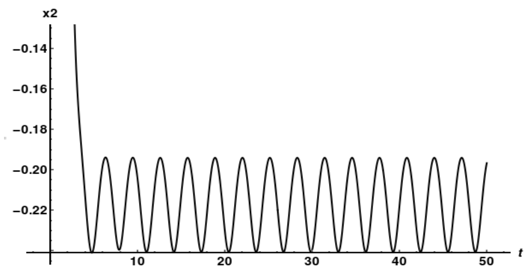

Fig.2: Graph of x_2 against t.

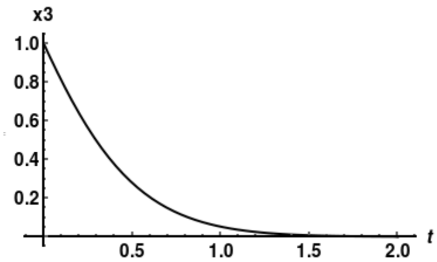

Fig.3: Graph of x_3 against t.

Its graph shows the convergence of the total concentrations of the populations of the three species to the optimal concentrations.

CONCLUSIONS

1. The coexistence of different species where prey and predators are present, their behavior in general has a cyclic shape, which makes one think that a better modeling of the process is by means of a periodic system in general.
2. Theorem1 allows to reduce the periodic system in general in a system where the matrix of the linear part has constant coefficients and the non-linear part of the system with periodic coefficients which simplifies analysis.
3. Theorem2 gives necessary and enough conditions so that the total concentrations of the populations of the three species converge to the optimal values of the populations; indicating the cases in which it is divergent and when it has no real meaning.
4. The examples shown corroborate the results shown theoretically, because by fulfilling the conditions of the theorem, the convergence to the optimum values persists.

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